Ocean wave energy, solar radiation, and characteristic times on the back of a Purcell envelope

Yoav Green, Emil J. Millet, and James P. Butler

Citation: American Journal of Physics **87**, 693 (2019); doi: 10.1119/1.5119684
View online: https://doi.org/10.1119/1.5119684
View Table of Contents: https://aapt.scitation.org/toc/ajp/87/9
Published by the American Association of Physics Teachers

LETTERS TO THE EDITOR

The downloaded PDF for any Letter in this section contains all the Letters in this section.

Letters are selected for their expected interest for our readers. Some letters are sent to reviewers for advice; some are accepted or declined by the editor without review. Letters must be brief and may be edited, subject to the author's approval of significant changes. Although some comments on published articles and notes may be appropriate as letters, most such comments are reviewed according to a special procedure and appear, if accepted, in the Notes and Discussions section. (See the "Statement of Editorial Policy" at http://ajp.dickinson.edu/edPolicy.html.) Running controversies among letter writers will not be published.

Ocean wave energy, solar radiation, and characteristic times on the back of a Purcell envelope

Yoav Green, Emil J. Millet, and James P. Butler
Department of Environmental Health, Harvard T.H. Chan School of Public Health, Boston, Massachusetts 02115

(Received 10 July 2019; accepted 10 July 2019)

https://doi.org/10.1119/1.5119684

Precious gems are precious precisely because they are rare. It is a great pity when they are lost or buried or forgotten. Such gems are the problems of the "Back of the Envelope" type that have appeared in this journal. These first appeared in the series posed by Edward Purcell over the years 1983–1988.[1] (See URL link to problems and solutions.[2]) Problems of a similar style, the "Search for simplicity problems," by Victor Weisskopf[3] appeared in 1985 and 1986. It is wonderful that since December 2017, the American Journal of Physics again has posed such problems, this time in the "Back of the Envelope" column written by Sanjoy Mahajan.[4] These problems are like peep-holes into the physical and mathematical universe, giving a tiny view, but if you move your head about, a much wider view is revealed. The problems span numerous disciplines where the length and time scales vary from the atomic to the cosmic. They are not only a delight but also solving them puts one in the company of people who think deeply about the essential physics of a host of problems. This is a nontrivial exercise; one needs to bring together all that one knows and distill the key features.

We focus in this letter on the problems of Purcell, which we recommend to everyone; one cannot be less than enlightened by both the apparent simplicity of the problem, and the depth of thinking that lies behind the back of the envelope. All physicists, young and old, should read and ponder these. You will be the better because of it. We would mourn if these were lost to history.

It is also true that the greatest among us occasionally makes an arithmetical error of substance. Such is the case for Problem 3 of November 1983, in which Purcell asks "In order of magnitude, the energy stored in ocean waves is as much as the earth receives from the sun in what length of time?" The published solution is off by 10^3.

He gives the correct expression for the total wave energy per unit ocean area, $\rho g h^2/8$, but for 2 m high waves, this gives roughly 5 kJ/m^2 rather than 5 J/m^2. This, together with assuming 1/2 the Earth is covered with oceans, and 1/4 of the Earth intercepts solar radiation of 1 kW/m^2 yields a time scale of $\frac{1}{2}5\,\mathrm{kJ\,m^{-2}}/\left(\frac{1}{4}1\,\mathrm{kW\,m^{-2}}\right) \approx 10\,\mathrm{s}$ rather than 10 ms. We note that if one crudely takes a first order linear ODE (with time constant τ) for energy damping from whatever mechanisms, and energy growth from solar power, the steady state solution is exactly this same ratio, namely, τ being the ratio of energy to input power. This argues that 10 s is an order of magnitude estimate for the characteristic damping time.

We thank the editor, Richard Price, for suggesting the damping time interpretation, for reminding us of the problems of Weisskopf and the recent resurrection of the Back of the Envelope in this journal.

[1]E. Purcell, Am. J. Phys. **51**, 107 (1983); *ibid.* **51**, 205 (1983); F. Duarte, Am. J. Phys. **53**, 615 (1985); E. Purcell, Am. J. Phys. **55**, 680 (1987); E. Purcell, Am. J. Phys. **56**, 12 (1988); *ibid.* **56**, 392 (1988).

[2]See <http://web.mit.edu/rhprice/www/Readers/backEnv.html> for all problems and solutions of Purcell's problems.

[3]V. F. Weisskopf, Am. J. Phys. **53**, 19–20, 109–110, 206–207, 399–400, 522–523, 618–619, 814–815, 940–942, 1140–1141 (1985); **54**, 13–14, 110–111 (1986).

[4]Sanjoy Mahajan, Am. J. Phys. **85**, 959–961 (2017).

